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**RADIATION BY A LINEAR ARRAY OF
HALF-WIDTH LEAKY-WAVE
ANTENNAS (PREPRINT)**

**Daniel Killips, Joshua Radcliffe, Leo Kempel,
and Stephen Schneider**



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Radiation by a Linear Array of Half-Width Leaky-Wave Antennas

Daniel Killips¹, Joshua Radcliffe², Leo Kempel¹ and Stephen Schneider²

Abstract—Leaky-wave antennas are interesting apertures for a variety of applications due to their low profile and wide bandwidth. They are inherently traveling wave antennas, and hence are best suited for end-fire applications. A new type of leaky-wave antenna, the half-width leaky-wave antenna (HWLW), has been recently investigated and found to have similar radiation properties as its full-width leaky-wave (FWLW) counterpart, but only requiring half the transverse dimension. In addition, the feeding mechanism for a HWLW antenna is considerably simplified compared to the FWLW antenna. This paper discusses arraying these antennas to provide both increased gain and scanning capability. It will be seen that arraying HWLW antennas is more complex than its narrowband counterpart, the patch antenna.

Index Terms—microstrip, leaky-wave, transverse resonance, traveling wave antenna, wide bandwidth, array.

I. INTRODUCTION

MICROSTRIP leaky-wave antennas have both low profile and wide bandwidth features. The thickness of a microstrip leaky-wave antenna is no greater than that of the common patch antenna albeit with substantially more bandwidth. Leaky-wave antennas have been extensively studied by Oliner [1], Menzel [2], Lee [3], and Lin [4-6] among others. The radiating mode for all of these cases is the EH_1 mode, e.g. the first higher-order mode. One of the major issues with operating a microstrip antenna in the first leaky-wave mode involves preferential excitation of that mode (as compared to the fundamental, non-radiating EH_0 mode). Menzel [2] utilized a periodic array of slots to suppress the fundamental mode while Lin *et al.* [4] utilized a more complex two-port feeding structure. All of the antennas examined in these previous papers were full-width leaky-wave (FWLW) microstrip antennas.

An alternative is the half-width leaky-wave (HWLW) antenna extensively studied by Thiele *et al.* [7-9]. This antenna is formed by placing a shorting ridge, between the microstrip and ground, along one long edge of the antenna. The other long edge will support a magnetic current wall that is

responsible for radiation. Full-width microstrip leaky-wave antenna arrays have been extensively studied [10-11]. For HWLW antennas, due to the relatively strong coupling between the array elements via the leaky-wave mode, it is useful to utilize full-wave computational electromagnetics (CEM) solvers rather than to rely on traditional antenna array theory that neglects coupling. Since the HWLW antenna occupies half the surface area of a conventional FWLW antenna, the potential for closer spacing between antenna elements is attractive for array design.

The HWLW antenna by itself has a major advantage over the FWLW antenna, relatively simple feed and loading. One of the major challenges of the FWLW antenna is suppression of the EH_0 mode. Clearly, if the antenna is operated at a frequency above the cut-off frequency for both the EH_0 and EH_1 modes, then both modes can be excited. Previous papers have discussed various methods for suppressing the lower-order mode in preference to the leaky-wave mode [2,4-6,11]. For the HWLW antenna, due to the presence of the shorting wall along one long edge, the lower order mode is automatically suppressed. The driving point impedance can be approximated using an open waveguide model [6] along with estimated wavenumbers using the Transverse Resonance Method [3]. Using this information, both the feed location and the location of a lumped load can be estimated. The purpose of the lumped load is to suppress the backward traveling wave that will exist if no load is used.

In this present paper, the radiation properties of an array of HWLW antennas is examined using a finite element-boundary integral (FE-BI) model. The antenna feeds, loads, and geometrical features will be modeled within the context of the FE-BI model. In addition, since this is a full-wave model, the coupling between elements attributed to the leaky-wave mode will be included. Radiation properties will be examined using the model.

II. MICROSTRIP LEAKY-WAVE ANTENNAS

A. Leaky-wave Antenna Theory

The fundamental mode of a microstrip line, the so-called EH_0 mode, is not a radiating mode (hence the popularity of microstrip transmission lines). The electric and magnetic fields for this mode are shown in Figure 1. Radiation from such a structure can be represented by two long magnetic

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current walls (for the FWLW antenna) separated the width of the microstrip. Since these current sources are out-of-phase, the co-polarized radiated field in a plane bisecting the width of the antenna is identically zero. Therefore, as an end-fire antenna, such a microstrip is unsuitable.

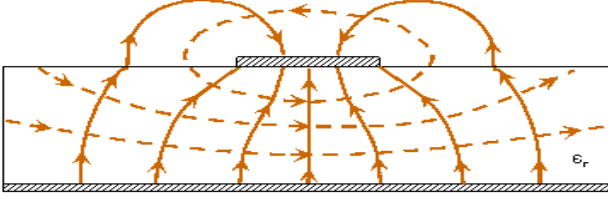


Figure 1. Field diagram for the EH_0 mode (E-field = solid, H-field = dashed).

Rather, higher-order modes must be preferentially excited to realize a radiating traveling wave structure. The first higher-order mode, the EH_1 mode, is one such radiating mode. This mode (shown in Figure 2) exhibits electric field odd symmetry

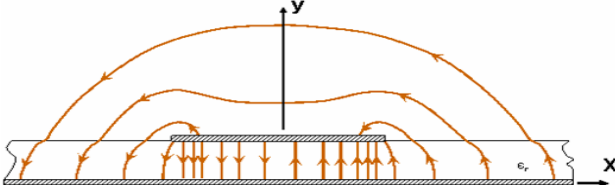


Figure 2. E-field diagram for the EH_1 mode

about the axial centerline of the antenna as compared to the even symmetry of the EH_0 mode. The radiating magnetic currents are now in-phase and hence radiate along the axis of the antenna.

The current on either wall can be represented as

$$\mathbf{M}(x, y, z) = \pm \hat{z} A_{\pm} e^{-\left(\frac{\alpha(z,f)}{k_0}\right)k_0 z} e^{-j\left(\frac{\beta(z,f)}{k_0}\right)k_0 z} \quad (1)$$

where the attenuation term $\alpha(z,f)$ and the propagation term $\beta(z,f)$ are in general a function of both position and frequency, k_0 is the free-space wavenumber, and the wave coefficients (A_{\pm}) are associated with the two magnetic wall currents at $x = \pm w/2$ where w is the width of the full-width microstrip. The attenuation and propagation terms for an axially invariant structure can be determined using the Transverse Resonance Method (TRM) [3]. Once the propagation parameters are known, the driving point impedance for a semi-infinite line can be estimated using an open waveguide model, viz. [5-6]³ as

$$Z_w = 8Z_0 \sin^2\left(\frac{\pi y}{w_{\text{eff}}}\right) \frac{k_0 h}{k w_{\text{eff}}} \sqrt{\frac{\mu_r}{\epsilon_r}} \quad (2)$$

The input impedance for a finite, loaded leaky-wave antenna can be estimated using the impedance transformation

$$Z_{\text{in}} = Z_w \left[\frac{Z_L + Z_w \tanh(\gamma L)}{Z_w + Z_L \tanh(\gamma L)} \right] \quad (3)$$

where Z_L is the load impedance (for this paper, $Z_L = 50\Omega$) and $\gamma = jk = j(\beta - j\alpha)$.

In (2), the effective microstrip width is given by Wheeler's approximation [12]

$$w_{\text{eff}} = h \left\{ \frac{w}{h} + \frac{2}{\pi} \ln \left[2\pi e \left(\frac{w}{2h} + 0.92 \right) \right] \right\} \quad (4)$$

Use of (2)-(4), with the propagation parameters provided by TRM, allows determination of the appropriate feed and load locations along the width of the strip.

Assuming a single radiating current of the type represented in (1), hence a leaky-wave antenna that is perfectly terminated but aligned parallel to the x -axis and centered on the origin, the radiated electric field in the far-zone (with the spherical wave assumed and suppressed) is given by

$$\mathbf{E}(\theta, \phi) \sim \frac{k_0 I_0}{4\pi L} \frac{\sinh\left(\frac{\chi L}{2}\right)}{\left(\frac{\chi L}{2}\right)} \left[\hat{\theta} \sin \phi + \hat{\phi} \cos \theta \cos \phi \right] \quad (5)$$

where $\chi = jk_0 [\sin \theta \cos \phi - k/k_0]$ and I_0 is the strength of the current. If a linear array of uniformly spaced, with a separation of d , similar elements is aligned along the y -axis, the array factor will be given by

$$AF = \sum_{m=-M}^M e^{jk_0 (md) \sin \theta \sin \phi} \quad (6)$$

Obviously, use of (5) and (6) assumes no coupling between array elements.

B. Example: Leaky-wave Antenna on Duroid

As an example, consider a leaky-wave antenna printed on Duroid 5870 (31 mils thick, $\epsilon_r = 2.33$, $\tan \delta = 0.0005$). The full-width strip width is 15 mm while the half-width strip width is 7.5 mm. The strip is taken to be 190 mm long for the simulations and measurements and it is infinite for the TRM analysis. The propagation terms, as determined using the transverse resonance method, are shown in Figure 3. Traditionally, the leaky-wave region of operation is defined between the frequency such that $\alpha = \beta$ to the frequency such that $\beta = k_0$; hence, for the example presented, approximately from 6 GHz to 8 GHz. Using this information, along with (2) and (3), the driving point impedance for a 190 mm realization of the antenna is shown in Figure 4 where the feed point is taken at the transverse midpoint of the HWLW antenna and a 50Ω lumped load is also placed along the midline of the antenna at the opposite end from the feed.

³ There is a typographical error in the expression in [6] where the sine function should be squared as shown in (2). The expression is correct in [5].

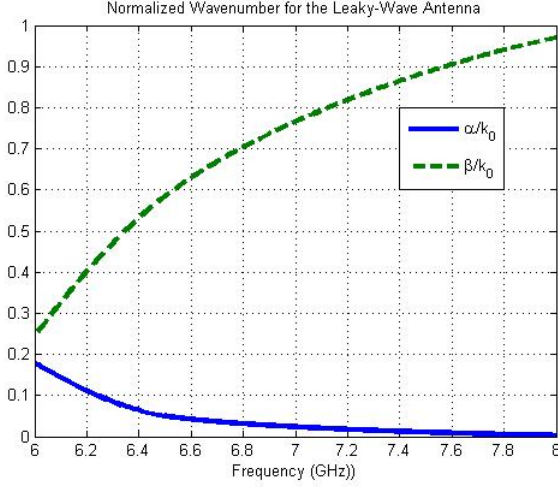


Figure 3. Normalized attenuation and phase constant for a leaky-wave antenna.

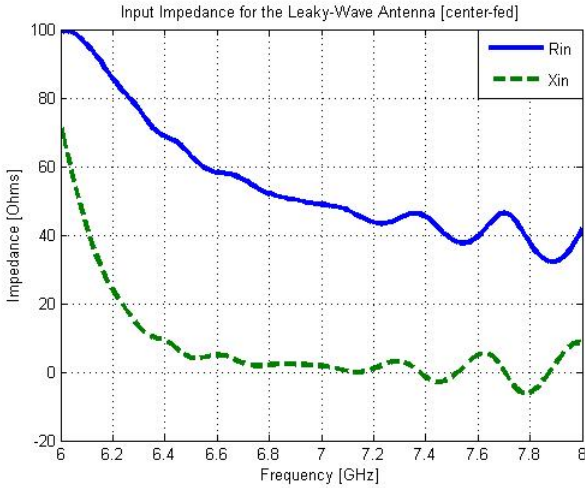


Figure 4. Input impedance at the center of the half-width strip width for this leaky-wave antenna.

As can be seen, the impedance seen at the input port of the antenna is dispersive and hence a perfect match to a 50 Ω line is not possible across the entire operational bandwidth. In addition, as noted in [10-11], the traveling wave fields guided by a microstrip structure couple rather strongly to the leaky-waves emanating from neighboring microstrips causing deviation from theoretical predictions for both the wavenumber, driving point impedance, and consequently, the radiated field. Hence, use of a full-wave computational method is desirable for designing leaky-wave arrays.

III. FINITE ELEMENT-BOUNDARY INTEGRAL MODEL

The hybrid finite element-boundary integral (FE-BI) method is widely understood to be an accurate and efficient method for modeling planar, microstrip apertures. The assumed geometry is a cavity-backed aperture recessed within an infinite metallic plane. The FE-BI formulation is an integro-differential equation representing a weak enforcement of the relevant boundary conditions applied to the vector wave equation. This FE-BI expression is given as

$$\begin{aligned} & \int_V \left[(\nabla \times \mathbf{W}_i) \cdot \bar{\bar{\mu}}_r^{-1} \cdot (\nabla \times \mathbf{E}^{\text{int}}) - k_0^2 \mathbf{W}_i \cdot \bar{\bar{\epsilon}}_r \cdot \mathbf{E}^{\text{int}} \right] dV + \\ & k_0^2 \int_S \int_S \mathbf{W}_i \cdot \left[\hat{\mathbf{z}} \times \bar{\bar{G}}_{e2} \times \hat{\mathbf{z}} \right] \cdot \mathbf{E}^{\text{int}} dS' dS = \\ & -jk_0 Z_0 \int_V \mathbf{W}_i \cdot \mathbf{J}^{\text{imp}} dV \end{aligned} \quad (7)$$

where \mathbf{W}_i is a vector testing function, $\bar{\bar{G}}_{e2}$ is the half-space dyadic Green's function, and \mathbf{J}^{imp} is an impressed current used to represent the antenna feed. The material in the computational volume, e.g. the antenna cavity, is represented by the relative permittivity tensor ($\bar{\bar{\epsilon}}_r$) and the relative inverse permeability tensor ($\bar{\bar{\mu}}_r^{-1}$). The FE-BI formulation is reduced to a linear system via Galerkin's procedure resulting in the following linear system

$$\begin{bmatrix} \mathbf{Y}_{ij}^{\text{FE}} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{E}_j^{\text{S}} \\ \mathbf{E}_j^{\text{V}} \end{Bmatrix} + k_0^2 \begin{bmatrix} \mathbf{Y}^{\text{BI}} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{E}_j^{\text{S}} \\ \mathbf{E}_j^{\text{V}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_i^{\text{int}} \end{Bmatrix} \quad (8)$$

In this, the unknown coefficients associated with the expansion functions (\mathbf{W}_j) are represented by (\mathbf{E}_j). For the unknown coefficients, (\mathbf{E}_j^{S}) represents unknowns associated with the surface while (\mathbf{E}_j^{V}) represent unknowns not associated with the surface. This system can efficiently be solved, both in terms of memory consumption and wallclock time, using an iterative solver such as the biconjugate gradient method [14].

By utilizing a fast Fourier transform (FFT) method to accelerate the computation of matrix-vector products in an iterative solution of the FE-BI linear system, simulation of a large antenna is feasible within practical design time limits. Hence, brick elements were used to discretize the computational volume [13]. The shorted side of the HWLW antenna is modeled using an infinitesimally thin perfectly conducting wall running from one end of the strip to the other end. The lumped loads were modeled as infinitesimally thin loads placed along the shorted edge of the half-width antenna. The feeds are modeled likewise as infinitesimally thin probe feeds placed along the shorted edge of the antenna opposite the loads on the mid-line of the antenna.

In addition, the computer program was parallelized using OpenMP [15] and run on a dual-core AMD 64-bit system with a total of four cores per node. The program has also been used on an SGI Altix 3700 Bx2 using 64-bit Itanium processors [16]. The primary routine that was parallelized was the iterative matrix solvers (the biconjugate gradient and transpose-free QMR algorithms are implemented) and fast Fourier transforms. For this work, the BiCG algorithm [14] was used.

IV. COMPUTED DATA FOR A FIVE ELEMENT LINEAR ARRAY

To validate the computational model discussed above, an experiment was conducted in a compact anechoic chamber using a single HWLW antenna. The shorting wall in the half-width antenna was approximated using shorting pins that were 5 mm apart running the length of the antenna strip. The ports were attached to an HP-8510 vector network analyzer. A representative radiation pattern, comparing computed and measured data, is shown in Figure 6.

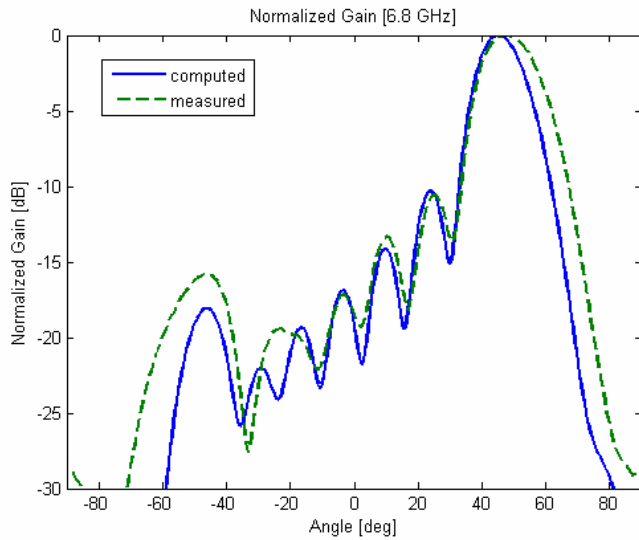


Figure 6. Comparison of computed and measured radiation patterns from a half-width, leaky-wave antenna at 6.8 GHz.

The agreement is reasonably good considering that the shorting wall was not solid, the FE-BI model used a probe feed, and the actual groundplane was finite while the modeled one was infinite.

Next, the FE-BI model was used to determine the impact of leaky-wave coupling on the fields supported by adjacent microstrip elements. Figure 7 illustrates a representative coupling between a driven element (the lower one in the figure) and an unfed element that is terminated on both ends of the antenna with 50Ω loads and 1.95 cm (center-to-center) away from the lower element. The simulation was conducted with a single cavity containing both the driven element and the passive element. In this manner, coupling between elements can occur via substrate, across the surface of the aperture, and via space-wave mechanisms. For an aperture where each element is contained within individual cavities, coupling via the substrate is suppressed. Since this is assumed to be the dominant mechanism, since the EH_1 mode is a leaky-wave mode, this geometrical arrangement is appropriate.

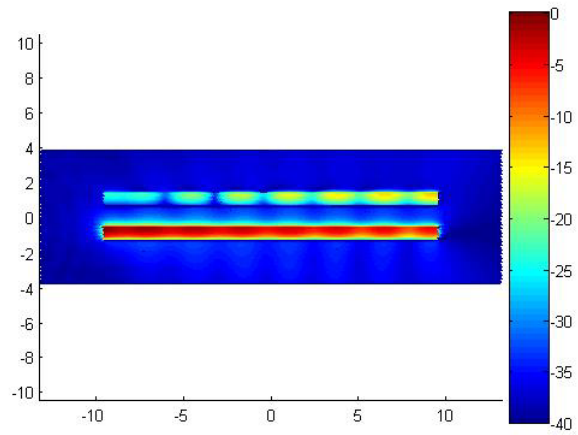


Figure 7. Normal fields in cavity for driven lower element and passive upper element.

It is clear that the leaky-wave from the driven lower element is coupled into the upper element. Notice that the fields in the upper element are strongest in the end of the element opposite the driving-point in the lower element; a phenomena expected from leaky-wave coupling since the driven leaky-wave will have a directional component from left to right in the figure. To assess the impact of this coupled field to the radiating current (and hence the fidelity of the radiation pattern to predictions based on a single traveling wave), the coupling ratio was determined for various separations. The coupling ratio is defined as the ratio of the maximum field strength in the region of the passive antenna to the maximum field strength in the region of the driven element. Hence, it is a measure of how strong one leaky-wave couples, and hence corrupts, the leaky-wave propagating in another microstrip. As a function of the center-to-center separation distance, the coupling between these elements at 7 GHz is given in the Figure 8. As can be seen, the coupling remains fairly strong even out to a distance of 6.45 cm. For 7 GHz, the free-space wavelength is 4.29 cm and hence a separation of 6.45 cm would result in grating lobes. This result enforces the assumption made previously that to accurately assess the radiation properties of a leaky-wave array, either the rather complex formulation presented in [11] or a full-wave model such as the FE-BI method must be used.

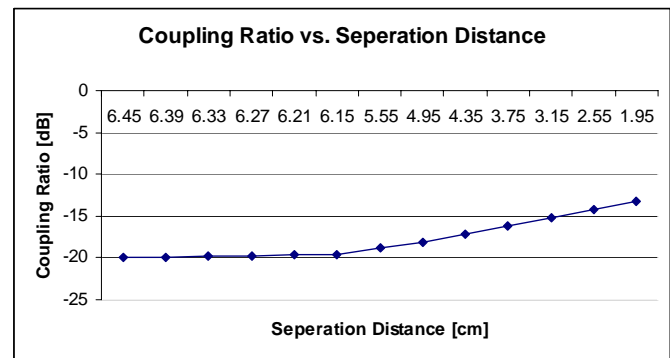


Figure 8. Coupling ratio between elements vs. separation distance.

A five-element array was simulated using the FE-BI model and having a separation of 1.2 cm between elements (e.g. less than $1/3^{\text{rd}}$ of the free-space wavelength to avoid grating lobes). The theoretically predicted radiation patterns based on (5) and (6) – e.g. the normalized radiation intensity [17] – is shown in Figures 9-10 for a uniformly illuminated array.

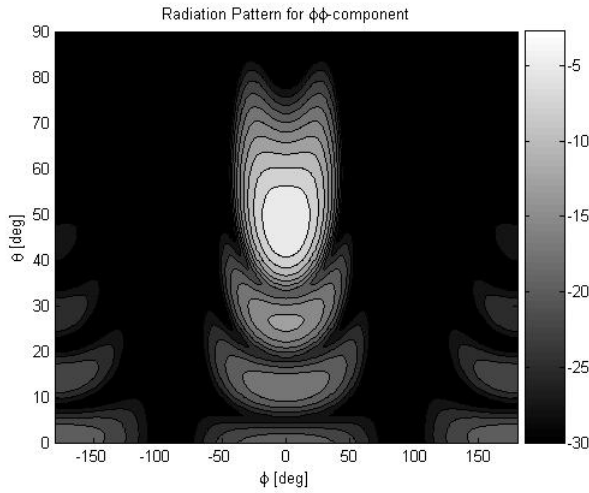


Figure 9. Theoretical ϕ - ϕ radiation pattern for a uniformly illuminated array.

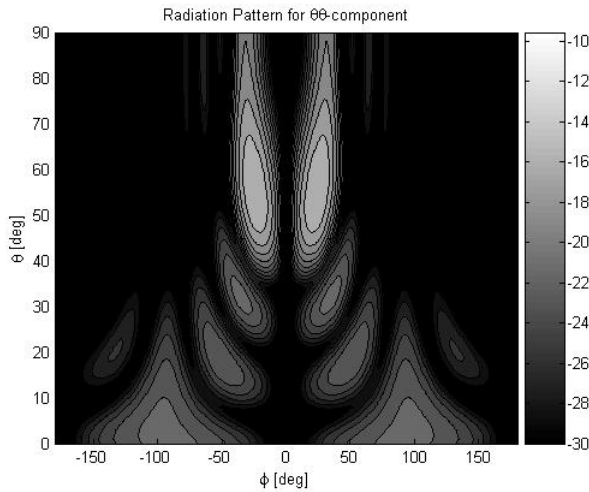


Figure 10. Theoretical θ - θ radiation pattern for a uniformly illuminated array.

The corresponding patterns as computed by the FE-BI model are shown in Figures 11-12, respectively.

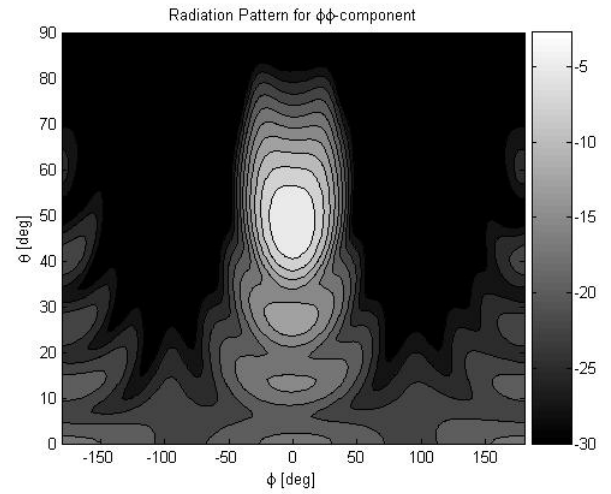


Figure 11. Numerical ϕ - ϕ radiation pattern for a uniformly illuminated array.

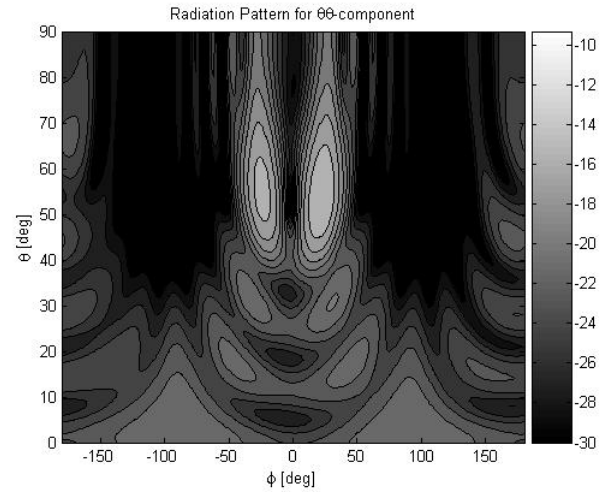


Figure 12. Numerical θ - θ radiation pattern for a uniformly illuminated array.

For all of the contour plots, each contour represents 3 dB of change. As seen by comparing Figure 9 to Figure 11 and Figure 10 to Figure 12, the agreement is quite good with the exception of the somewhat diffused sidelobes for the numerically computed results. The reason is presumably due to the leaky-wave coupling that is neglected in the theoretical results.

Another example is the same array; however, the main beam is now steered 30 degrees from the array axis normal. The theoretical results are shown in Figures 13-14 and the numerical results are shown in Figures 15-16, respectively.

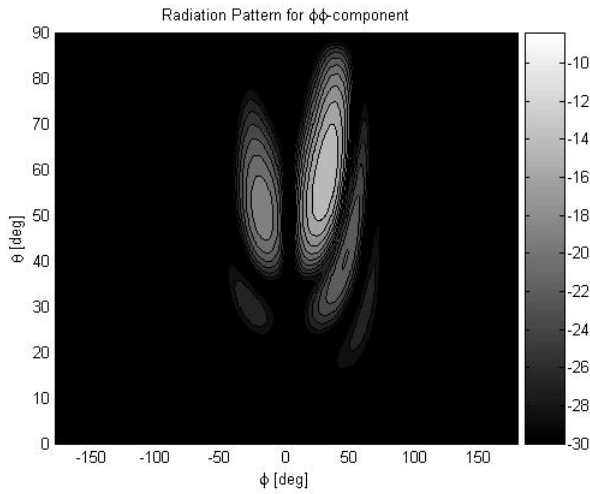


Figure 13. Theoretical ϕ - ϕ radiation pattern for an array steered 30 degrees off the x-axis.

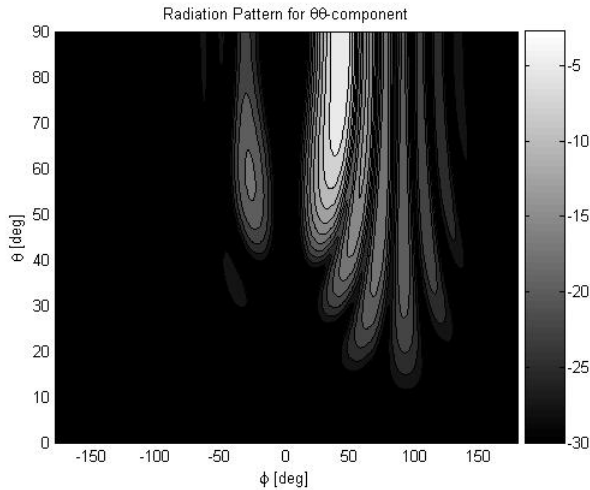


Figure 14. Theoretical θ - θ radiation pattern for an array steered 30 degrees off the x-axis.

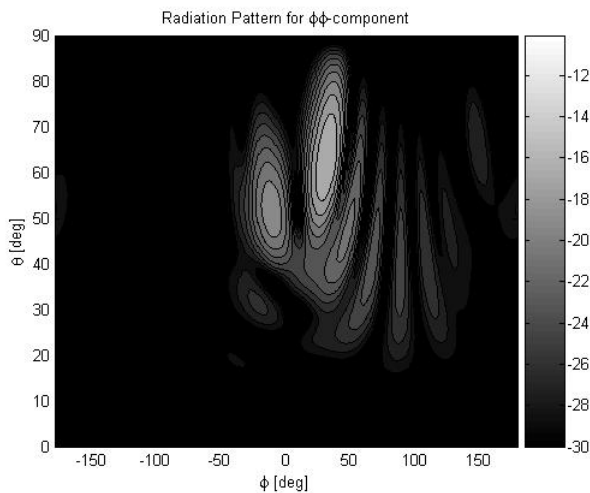


Figure 15. Numerical ϕ - ϕ radiation pattern for an array steered 30 degrees off the x-axis.

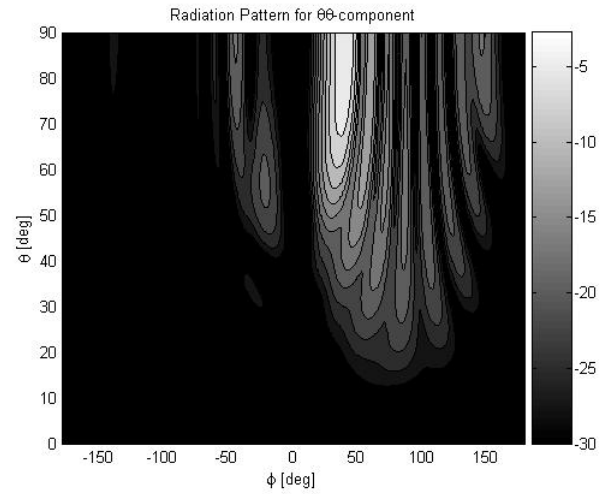


Figure 16. Numerical θ - θ radiation pattern for an array steered 30 degrees off the x-axis.

Again, the agreement between the theoretical and numerical results is quite good with the exception of higher sidelobes for the numerical results.

V. CONCLUSIONS

Microstrip leaky-wave antennas offer an attractive end-fire aperture that is thin and easy to manufacture. The half-width leaky-wave antennas investigated herein have a particularly simplified feed structure compared to the more traditional full-width leaky-wave antennas. An array of these antennas was investigated using a hybrid finite element-boundary integral method. Strong coupling between adjacent elements was observed and quantified. Nevertheless, the radiation patterns between the theoretical and numerical results agreed quite well except for the sidelobes. It is assumed that the leaky-wave coupling between elements perturbs the radiating magnetic current for each element causing changes in the sidelobes. Hence, for applications requiring tight sidelobe control, use of a full-wave analysis method is essential.

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